

## Syllabus

### Subject

<b>Subject / Group</b>	11001 - Dynamical Systems and Chaos / 1
<b>Degree</b>	Master's in Physics of Complex Systems
<b>Credits</b>	6
<b>Period</b>	1st semester
<b>Language of instruction</b>	English

### Professors

Lecturers	Office hours for students					
	Starting time	Finishing time	Day	Start date	End date	Office / Building
Manuel Alberto Matias Muriel <i>Responsible</i>	14:00	15:00	Tuesday	01/10/2019	29/05/2020	Desp. 211 (Edif. Instituts Universitaris)
Konstantin Klemm -	14:30	16:00	Thursday	12/09/2019	04/06/2020	102 / IFISC

### Context

#### INSTRUCTORS

Manuel Matias holds a PhD in Physics (1997) and has five 6-year terms ("sexenios") recognized by the ANEP. He has research experience in several aspects of Nonlinear Dynamics, like Chaos Theory, coupled oscillators, synchronization, dynamics of localized structures in extended media, etc. More recently, he is working on the dynamics of biological systems.

Konstantin Klemm holds a PhD in Physics (Niels Bohr Institute, 2004). He has research experience in Nonlinear Dynamics of Network Systems, Applications of Dynamical Systems in Biology and further related topics.

#### COURSE

The aim of this course is to train potential researchers in the study of nonlinear dynamical systems. The course concentrates on low-dimensional dynamical systems, systems of differential equations and maps, analyzing the different solutions that are possible depending on the dimensionality of the phase space on which the dynamical system lives and also their stability, together with the bifurcations in which these solutions appear or lose stability and viceversa. The analysis concentrates on stationary and periodic solutions, but an introduction is provided to systems exhibiting chaotic behavior, and also to some of the consequences of this behavior, like the fractal character of strange attractors. The course presents also modern tools useful to analyze these behaviors: Lyapunov exponents and dimensions. The course concludes with some applications: synchronization of coupled oscillators, excitability, phase space nonlinear time series analysis and systems with delay.

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The concepts and methods studied in this course are useful in other courses of the Master.

### Requirements

This is one of the compulsory courses of the Structural Module of the master in Physics of Complex Systems, that provides the foundations to analyze the behavior of deterministic dynamical systems, that are the backbone of complex systems, mainly mean-field descriptions.

The contents covered in this course are particularly relevant to those of the obligatory course (11004) "Pattern formation", that studies spatially extended dynamical systems, taught in parallel to this course. Some courses of the Specific Module, like (11009) "Spatiotemporal dynamics", (11018) "Turbulence and nonlinear phenomena in fluid flows" or (11012) "Modelling and dynamics of neural systems" extend or apply the contents of this course in different contexts.

### Recommended

It is recommended that the student has a basic knowledge of linear algebra, systems of linear ordinary differential equations and numerical integration of differential equations (Euler and Runge-Kutta methods).

### Skills

#### Specific

- \* E8: To know to characterize generic behavior of dynamical systems and their instabilities.
- \* E9: To know stability analysis techniques and know how to build bifurcation diagrams.
- \* E10: To know to characterize chaos and know how to calculate Lyapunov exponents
- \* E11: To know how to apply dynamical systems techniques to physical, chemical, biological and social systems.

#### Generic

- \* TG1: To be able to describe, both mathematically and physically, complex systems in different situations
- \* TG2: To acquire the capacity to develop a complete research plan covering from the bibliographic research and strategy to the conclusions.
- \* TG6: To acquire high power computation skills and advanced numerical methods capabilities in applications to problems in the context of complex systems.

#### Basic

- \* You may consult the basic competencies students will have to achieve by the end of the Master's degree at the following address: [http://estudis.uib.cat/master/comp\\_basiques/](http://estudis.uib.cat/master/comp_basiques/)

### Content

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### Range of topics

#### 1. Introduction

Phase Space, Existence and unicity of trajectories, Liouville theorem, Hamiltonian vs dissipative systems.

#### 2. One dimensional flows

Geometric representation. Fixed points. Potential representation. Stability analysis. Saddle-node bifurcation. Transcritical bifurcation. Pitchfork bifurcation. Normal forms. Bifurcation diagrams. Structural stability. Imperfect bifurcations and catastrophes.

#### 3. Two dimensional flows

Phase portraits. Fixed points. Stability. Forced damped oscillators. Limit Cycles. Index theory. Hopf bifurcation. Gradient systems. Lyapunov functions. Poincaré Bendixson theorem. Liénard Systems. Van Der Pol oscillator. Relaxation oscillations. Weakly nonlinear oscillators. Multiple time scale analysis.

#### 4. Three dimensional flows

Lorenz model. Simple properties of the Lorenz equations. Chaos on a strange attractor. Lorenz map. Exploring Parameter Space.

#### 5. One dimensional maps.

Fixed points and cobwebs. Logistic map. Numerics. Analysis. Chaos. Calculation of the Lyapunov exponent. Routes to chaos: period-doubling and intermittency. Universality. Feigenbaum's renormalization theory. Poincaré map. Floquet analysis.

#### 6. Fractals

Cantor set. Self-similarity. Dimension of a self-similar fractal. Dimension of real fractal sets: box counting, information and correlation dimensions. Generalized dimensions  $D_q$ . Lyapunov exponents. Kaplan-Yorke conjecture. Strange attractors.

#### 7. Entrainment and synchronization of oscillators

Quasiperiodicity. Circle map. 1:1 frequency locking. Rational lockings. Arnold tongues. Devil's staircase. Weakly coupled oscillators. Reduction to phase dynamics. Synchronization. Landau-Stuart oscillators. Oscillator death. Kuramoto model: Diversity, Order Parameter, Self-consistent solution.

#### 8. Excitability

Biological motivation. Active rotator. FitzHugh-Nagumo.

#### 9. Delayed systems

Delay in physical and biological systems. Fixed points. Stability analysis of 1-D linear DDEs: Hopf bifurcation. Mackey-Glass model.

#### 10. Non-linear time series analysis

Poincaré section. Fourier characterization. Embedding methods.

## Teaching methodology

In-class work activities (1.5 credits, 37.5 hours)



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Modality	Name	Typ. Grp.	Description	Hours
Theory classes	Theory classes	Large group (G)	Lectures explaining the theoretical concepts given by the professor.	28
Practical classes	Practical sessions	Large group (G)	Resolution of practical examples and questions.	5.5
Laboratory classes	Lab sessions	Medium group (M)	This activity aims at the visualization of the nonlinear phenomena in real experimental systems. Experiments will be performed in mechanical, electronic or chemical systems.	1
Assessment	Exam	Large group (G)	The student has to answer the questions of a written exam.	2.5
Assessment	Oral presentation	Large group (G)	Each student will be given an individualized assignment that covers several of the topics of the course. Besides a written report, the student has to give an oral presentation to the whole class.	0.5

At the beginning of the semester a schedule of the subject will be made available to students through the UIBdigital platform. The schedule shall at least include the dates when the continuing assessment tests will be conducted and the hand-in dates for the assignments. In addition, the lecturer shall inform students as to whether the subject work plan will be carried out through the schedule or through another way included in the Aula Digital platform.

### Distance education tasks (4.5 credits, 112.5 hours)

Modality	Name	Description	Hours
Individual self-study	Study and understanding theoretical concepts	This activity aims at the understanding of the theoretical concepts and techniques explained in the lectures	52.5
Individual self-study	Exercises	The student has to solve exercises assigned and present the solutions in both written form and orally to the rest of the class.	30
Group or individual self-study	Realization of the assignment	Students in teams must solve their assignment, prepare a report and organize an oral presentation.	30

### Specific risks and protective measures

The learning activities of this course do not entail specific health or safety risks for the students and therefore no special protective measures are needed.

### Student learning assessment

#### Frau en elements d'avaluació

In accordance with article 33 of Regulation of academic studies, "regardless of the disciplinary procedure that may be followed against the offending student, the demonstrably fraudulent performance of any of the

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evaluation elements included in the teaching guides of the subjects will lead, at the discretion of the teacher, a undervaluation in the qualification that may involve the qualification of "suspense 0" in the annual evaluation of the subject".

### Exam

Modality	Assessment
Technique	Objective tests ( <b>retrievable</b> )
Description	The student has to answer the questions of a written exam.
Assessment criteria	Accuracy of the answers. Clarity and quality of the explanations.

Final grade percentage: 50%with a minimum grade of 4

### Oral presentation

Modality	Assessment
Technique	Oral tests ( <b>non-retrievable</b> )
Description	Each student will be given an individualized assignment that covers several of the topics of the course. Besides a written report, the student has to give an oral presentation to the whole class.
Assessment criteria	Accuracy and quality of the work as well as the clarity in the oral exposition.

Final grade percentage: 10%with a minimum grade of 5

### Exercises

Modality	Individual self-study
Technique	Papers and projects ( <b>non-retrievable</b> )
Description	The student has to solve exercises assigned and present the solutions in both written form and orally to the rest of the class.
Assessment criteria	Accuracy of the results. Clarity and quality of the explanations and interpretation of the results. Quality of the written and oral presentations.

Final grade percentage: 20%with a minimum grade of 5

### Realization of the assignment

Modality	Group or individual self-study
Technique	Papers and projects ( <b>non-retrievable</b> )
Description	Students in teams must solve their assignment, prepare a report and organize an oral presentation.
Assessment criteria	Accuracy of the answers. Clarity and quality of the explanations.

Final grade percentage: 20%with a minimum grade of 5

## Resources, bibliography and additional documentation

The course follows the most clear textbook at the level of this course (Strogatz book). In addition, there is a complementary bibliography for some of the topics.

### Basic bibliography



## Syllabus

S.H. Strogatz, "Nonlinear Dynamics and chaos", Addison Wesley 1994 / Westview Press, 2000.

### Complementary bibliography

- E. Ott, "Chaos in Dynamical Systems", Cambridge University Press, 2nd edition, 2002 [mainly for chapters 5-7].
- K.T. Alligood, T.D. Sauer, and J.A. Yorke, "Chaos. An Introduction to Dynamical Systems", Springer, 1997 [mainly for chapters 5,7].
- A. Pikovsky, A. Politi, "Lyapunov exponents: A tool to explore complex dynamics, Cambridge University Press, 2016 [mainly for chapter 6].
- A. Pikovsky, M. Rosenblum, J. Kurths, "Synchronization: A universal concept in nonlinear sciences", Cambridge University Press, 2001 [for chapter 7].
- S.H. Strogatz, "From Kuramoto to Crawford", Physica D **143**, 1-20 (2000) [for chapter 7].
- J.D. Murray, "Mathematical biology", 3rd edition, Springer, 2003 [for chapter 8].
- T. Erneux, "Applied Delay Differential Equations", Springer, 2009. [mainly for topic 9].
- H.D.I. Abarbanel, "Analysis of Observed Chaotic Data", Springer, 1996 [mainly for topic 10].

